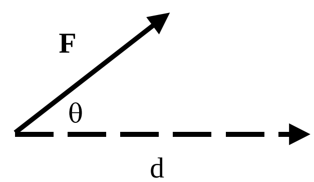
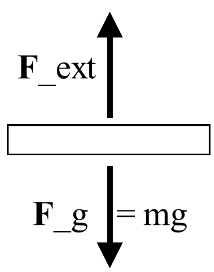
We've been talking about electric forces, and the related quantity
E = **F**/q, the E field, or "force per unit charge". In mechanics, after talking about forces, we moved on to work and energy.

<u>*Quick Review of work and energy:*</u>



The work done by a force, F, moving something through a displacement "d", is
Work = F*d, or more carefully, $W = F_{||} d \;=\; \mathrm{F\,d}\cos(\theta)$.

E.g. if you (an "external force") lift a book (at constant speed) up a distance d,



Newton II says **F**_net = m**a**,
i.e. **F**_ext - **F**_g = 0
(because, remember, if speed is constant => **a**=0)
or **F**_ext = mg.

You do work W_ext = F_ext*d = +mgd
(The + sign is because θ is 0 degrees, your force is UP, and so is the displacement vector)
The gravity field does W_field = -F_g*d = -mgd
(The minus sign is because θ is 180 degrees, the force of gravity points DOWN while the displacement vector is UP)

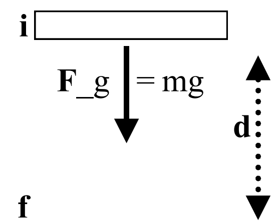
The NET work (done by all forces) is W_ext+W_field = 0, that's just the *work-energy principle*, which says W_net = ΔKE (=0, here)

You did work. Where did it go? NOT into KE: it got "stored up", it turned into *potential energy (PE).* In other words, F_ext did work, which went into *increased* gravitational potential energy.

For gravity, we defined this potential energy to be PE = mgy, so
Δ PE = mg(y_final - y_initial) = + mgd (=W_ext)

(The change in PE is all we ever cared about in real problems)

Now, let's *drop* the book, and see what happens.
There is no more "external force" touching the book (like "me" in the previous example), only gravity acts. (Neglect friction)



Energy conservation says

$$PE_i + KE_i = PE_f + KE_f, \quad \text{i.e.} \quad mgd + 0 \;=\; 0 \;+ \frac{1}{2} m v_f^2.$$

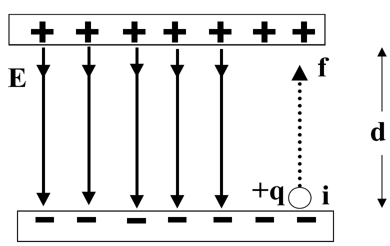
This formula gives a quick and easy way to find v_f.
The concept of energy, and energy conservation, is very useful!
Another way of rewriting that equation is

$$(PE_f - PE_i) + (KE_f - KE_i) = 0,$$

$$\text{i.e.} \quad \Delta PE \;+\; \Delta KE = 0, \qquad \text{or} \qquad \Delta E_{tot} = 0$$

(End of quick review of work and energy)

There is an electric "analogue" of the above examples:



Consider 2 charged parallel metal plates (called a "capacitor"), a fixed distance d apart.
Between the plates, **E** is uniform (constant), and points *from* the "+" *towards* the "-" plate.

Imagine a charge +q, initially located near the bottom plate.
The force on that charge is **F_E**=+q**E** (down, do you see why?).
(Let's totally neglect gravity here.)
Now LIFT "q" from the bottom to the top, at constant speed:

You do work W_ext = F_ext*d = +qEd
The *Electric* field does W_field = -F_E*d = -qEd.
(Do you understand those signs? Think about them!)

Just like the previous case: you did work, but where did it go?
As before, it didn't turn into KE, it turned into potential energy.
We say the charge's electrical potential energy has increased:
Δ PE = qE(y_final - y_initial) = + qE d (=W_ext)
(where y is the distance *above* the negative plate)

We lifted the charge from a region of LOW PE (near the "-" plate)
to a region of HIGH PE (near the "+" plate). (Note: "up" and "down" are *irrelevant* here, you could turn the picture on its side or even upside down. It's not gravity in this story, it's 100% electrical energy.)

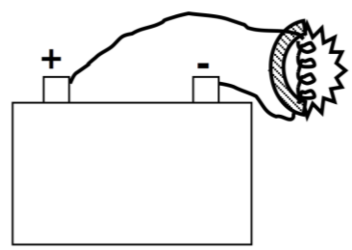
Just like we defined **E**=**F**/q (dividing out q gives force/unit charge)
let's now define something we call "electrical potential" or just "potential" = $\boxed{V = PE/q}$.

• Calling this quantity "potential" is really a pretty bad name, because this "potential" is quite DIFFERENT from "potential energy".
• Potential has units of [energy/charge] = [Joules/Coulomb] = J/C.
We call 1 J/C = 1 Volt = 1V
(People use the symbol "V" for the *unit* volt, as well as for the *quantity* itself. Another bad choice, but we have to live with it)

A change in potential is called a "potential difference",

$\Delta V = V_f - V_i = \Delta PE / q$, and from this we see $\boxed{\Delta PE = q\,\Delta V}$.

__Example__: A car battery maintains 12 V between the terminals. If the headlights contain a 36 W bulb, how much charge is the battery moving through the bulb each second? (And, how many electrons is that?)



__Answer__: 36 W = 36 Watt = 36 J/s. Each second 36 Joules of energy are dissipated in a bulb. This energy all comes from the loss of potential energy as charges flow from one terminal, through the bulb, to the other terminal. If a charge "q" drops 12V, the energy lost is $\Delta PE = q\Delta V$, or q*12V. Each second, 36 J are lost, i.e.
36 J= q*12 V, or q = (36 J)/(12 V) = (36 J)/(12 J/C) = 3 C.
That's a lot of electric charge being moved by a car battery!
The number of electrons going through the bulb each second is
3C/(1.6E-19 C/electron) = 2E19 electrons. (A heck of a lot)

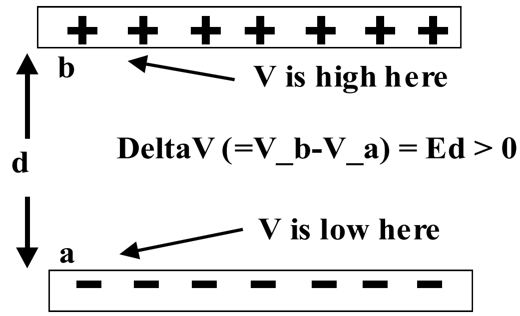
I was a little sneaky about signs (the charge of an electron is negative): just think about it.

Here's a related question for you: given that it's (negative) electrons that flow out of a battery, which way do they go? from the "+" terminal through the bulb to the "-", or the other way?

(The answer is from - to +. Electrons are repelled from "-", and attracted towards "+".)

For a parallel plate capacitor, we just found (two pages ago)
Δ PE = + qE d, so $\Delta V = \Delta PE / q = (qEd) / q = E d.$

Here's another sketch of a capacitor:



With gravity, you can choose to call "zero" potential energy wherever you want. You might choose sea level, or the tabletop, or the ground. It's the same story with electricity: you can pick any spot you want and call the electrical potential energy 0 there. We usually call this point "the ground"! Let's call point "a" in the diagram above "the ground" or "0 potential".

Now put a charge "+q" at the point "b" in that figure. It will have a potential given by V(at point b) = E*d.
It has a potential *energy* at point b of PE = +q*V(at b) = +qEd.

It has "+" potential energy there, which makes sense. It's like a pebble up in the air, it can do work, just let it go! (The upper plate repels a "+" charge, the lower plate attracts it: if you let it go it will run "downhill" in energy, from high potential to low...)

Notes:

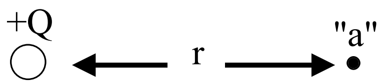
- **E** points from high V to low V *(always!)*

"+" charges want to head towards low V, if you'll let them
"-" charges want to head towards HIGH V, if you'll let them (!)

- We can talk about the “potential at a point”, or the “potential energy at a point”, but the numerical value depends on where we chose to call 0.

We can talk about “potential differences” between points, and then it does NOT matter where we chose to call 0!

What if you're near a point charge, Q, rather than a capacitor?



What's the potential at the point "a"?

1st, where do we want to call “0 potential”. It’s not so obvious here. A standard choice is “far away”, off at infinity. Out there, PE=0, V=0, seems reasonable!

Now we need to think about moving a test charge "q" from far away (where PE=0, V=0), to the point "a". Because the work you do bringing it from PE=0 to the point *is* precisely its potential energy!
(It's like how much work you do lifting a book from the ground, i.e. PE = 0, up to a height d: it's mgd, the final potential energy)

Now, Work = F*distance, and F=kQq/r^2. Unfortunately, this force *changes* as you move in from far away (r is changing). So, you really need calculus to figure out the work. The answer, though is very simple (and maybe you can even guess it, just multiply F*r...)

W_ext = k Q q/r (Notice, that's an r downstairs, not an r^2!)

So the PE at point "a" is exactly that, PE(at "a") = k Q q/r, or

V(at "a") = PE(at "a")/q = k Q/r .

(Note that we *chose* V=0 to be off at infinity, to get that formula.)

If Q and q are both "+", then PE=kQq/r >0. (This makes sense: two positive charges want to "fly apart", they'll DO work if you'll let them. The system has *positive* potential energy. Like a rock up in the air...)

Also, just like with capacitors, the potential V is big ("+") when you're near a positive charge Q. (the closer, the bigger.)

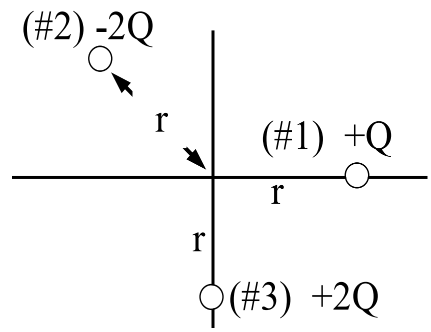
If Q and q are *opposite* signs, then PE<0. This is also correct: you would have to do work ON opposite charges to "pry them apart", the system has a negative potential energy! We might say the system is “bound”.

What if there are a bunch of point charges, what's the potential?

The voltage at any point is just the sum of the voltages arising from each of the individual particles (this is "superposition" again)

It's really quite *easy* to find the voltage at a point because of this!

Example: Three charges (#1, #2 and #3 with charges +Q, +2Q, and -2Q respectively) are arranged as shown. *What is the potential, V, at the origin?*



Answer:

V=V(from #1)+V(from #2)+V(from #3)

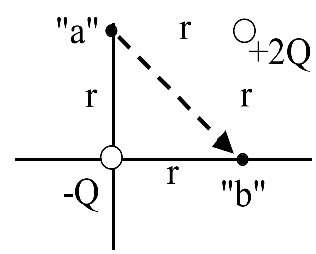
= kQ/r + k(-2Q)/r + k(+2Q)/r

= kQ/r.

(The answer is positive: if you put a positive charge there,
it would be happy to run away off to infinity if you'd let it)

The math is reasonably simple! No vectors or components to worry about - finding **E** at the origin would be a *lot* more trouble.

Example: A test charge "+q" is moved from point "a" to "b" in the figure. (There are two other charges present, -Q and +Q, fixed in position at the corners of a square, as shown.) *How much work does this take?*



Answer: $\boxed{W_{ext} = \Delta PE = PE_b - PE_a}$.

(Remember: at any point the potential energy is just PE= qV)

$$PE_b = +q \cdot V(\text{at b}) = +q \cdot \left(\frac{k(-Q)}{r} + \frac{k(+2Q)}{r}\right) = \frac{kQq}{r}(+1),$$

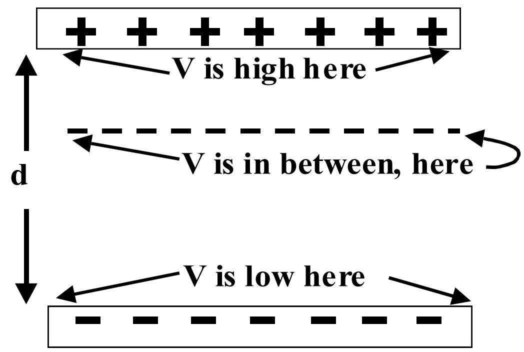
$$PE_a = +q \cdot V(\text{at a}) = +q \cdot \left(\frac{k(+2Q)}{r} + \frac{k(-Q)}{r}\right) = \frac{kQq}{r}(+1).$$

Subtracting, we find $W_{ext} = \Delta PE = PE_b - PE_a = 0$. It doesn't take any net external work at all.

(Depending on how you move, you might do some +work part of the way, and -work part of the way, but in the end, you do zero total work going from this particular "a" to "b".)

Trying to figure out the work by thinking of "force*distance" along the path would be HARD, because force changes all the time. Using voltages makes this much easier to figure out.

It's often useful to find *all* the points in a diagram that have the same voltage. E.g., consider a capacitor again.



Everywhere along the top surface, the potential is the same (V=Ed).

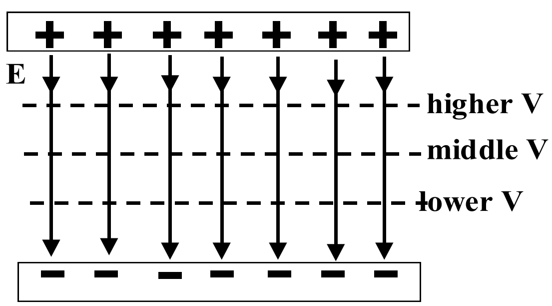
Everywhere along the bottom, the potential is the same (V=0).

And *every* point on that dashed line has the same potential. (something between 0 and Ed)

We call such a line an "equipotential" (equal potential) line.

If you move a test charge along an equipotential line (e.g. the horizontal dashed line in the figure) the potential is the *same* everywhere, so *no* work is required. It's like walking along a flat surface where there's no change in the gravitational potential. Or, e.g. *traversing* sideways on a ski slope.

There can't ever be a component of **E** parallel to an equipotential line. (if there *was* a nonzero **E**_parallel, you'd do work moving along it, since W = **F**_parallel*d = q **E**_parallel*d) This means that in drawings, **E** field lines are always perpendicular to equipotential lines.

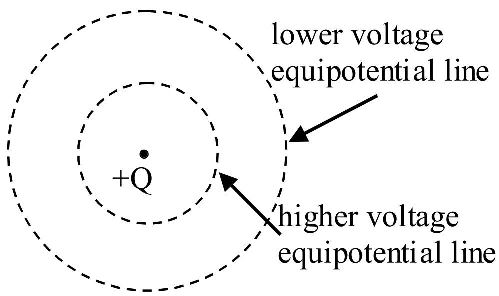


The equipotential lines are shown as "dashed" in this figure.
They're like contour lines on a topo map, which show "gravitational equipotential" lines. (Constant height on a topo means constant PE_grav).

Anywhere along a dashed line, the potential is constant.

- *Inside* any (static) conductor, we know **E**=0. That means no work is required to move charges around anywhere inside, or along the surface. So metals are equipotentials throughout their volume!
- Real life is 3-D, those “lines” are really “surfaces”...

More examples of equipotential lines:

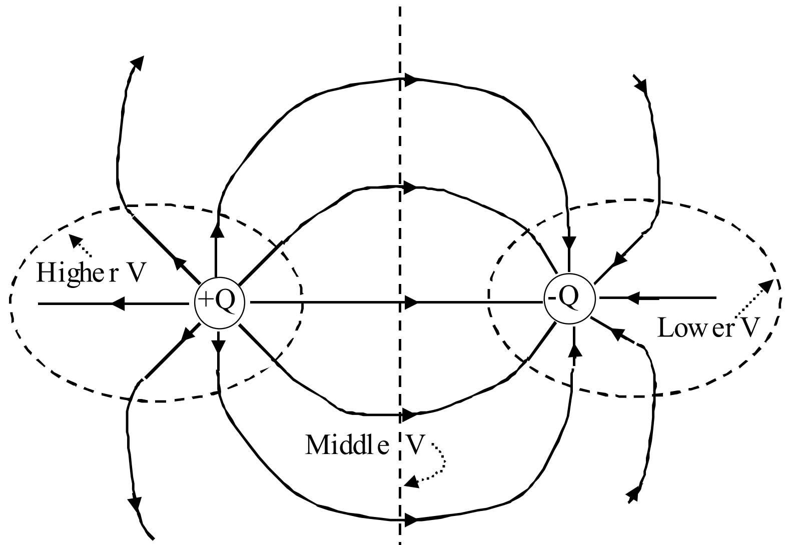


Remember, voltage near a charge +Q is given by V = kQ/r.

The farther away you get, the lower the voltage.

If "r" is fixed (i.e. a circle), V is constant.
(Again, in 3-D, these would be equipotential surfaces, rather than lines. In this case, they'd be spheres surrounding the charge Q)

Another example: here's a dipole, with a couple of equipotential lines shown. (Look back in Ch. 16, we sketched the electric field lines before. Now we're adding in the "equipotential surfaces)

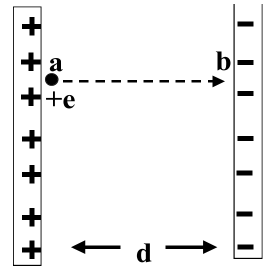


We usually choose to define that center equipotential line as V=0.
Remember, it's up to you to pick where V=0 is, and that line extends out to infinity. That's a pretty common choice. (Far away from everything, the potential is considered zero.)

To find the *numerical* value of the potential on one of those dashed surfaces, you'd do a quick calculation just like 2 pages ago - find the distance r1 to +Q, and r2 to -Q, and then V = kQ/r1 + k(-Q)/r2.
Anywhere on the center line, you're equidistant from both charges, so r1=r2, and the two terms cancel, V = 0. It's all consistent...

ENERGY, and units:

Example: Release a proton (charge +e), from point a in the figure. Suppose the voltage difference between the plates is 5000 V
(a typical voltage in e.g. a normal TV set.)
How fast is the proton going at point b?



Answer: I'd use conservation of energy here:

$$PE_i + KE_i = PE_f + KE_f, \quad \text{i.e.} \quad +eV_a + 0 = +eV_b + \tfrac{1}{2} m v_b^2$$

Solving for v_b we get

$v_b = \sqrt{2e(V_a - V_b)/m} = \sqrt{2(1.6 \cdot 10^{-19}\,\mathrm{C})(5000\,\mathrm{J/C})/(1.67 \cdot 10^{-27}\,\mathrm{kg})} = 10^6\ \mathrm{m/s}$ Can you *check* that the units worked out o.k.?)

Note that $V_a - V_b = +5000\ \mathrm{J/C}$, i.e. V_a is a HIGHER potential. However, the *change* in potential of the proton as it moves is $\Delta V = V_b - V_a = -5000\ \mathrm{J/C}$. Think about the signs: objects spontaneously move to LOWER potential energy if they can. That means "+" objects like to go to lower *potential (i.e. lower voltage).*

What was the final KE of the proton in this example? It's easy enough to find, using conservation of E:

$$KE_f = KE_i + PE_i - PE_f = 0 + eV_a - eV_b = e \cdot (5000V)$$
$$= 1.6 \cdot 10^{-19}\,\mathrm{C} \cdot (5000\,\mathrm{J/C}) = 8 \cdot 10^{-16}\,\mathrm{J}$$

Many people prefer to change units here, like converting 2.54E-2 m to 1 inch (metric to non-metric) We can define a new *unit* of energy:

1 eV = 1.6E-19 J

The "eV" is also called an "electron Volt", but it is NOT a volt (which is a unit of potential, or J/C, recall) It's just a name, "ee-vee"!

The eV is defined so as to be the energy loss of a particle of charge "e" (like a proton) dropping across 1 Volt of potential.
Since energy change is q*Delta V, this is an energy change of
(+e)*(1 V) = (1.6E-19 C)*(1 J/C) = 1.6E-19 J, just like we said.

In the little problem at the top of the page we could've done it without a calculator if we'd used eV's instead of J. Namely,

$$KE_f = KE_i + PE_i - PE_f = 0 + eV_a - eV_b = e \cdot (5000\,V) = 5000\,eV$$

You can check that this agrees by doing a simple unit conversion:

$5000\,eV = 5000\,eV \; * \dfrac{1.6 \cdot 10^{-19}\,\mathrm{J}}{1\,eV} = 8 \cdot 10^{-16}\,\mathrm{J}$, which is what we got above.